MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023 (TERM ID 2260)

ETM7166 – DIGITAL SIGNAL PROCESSING SYSTEMS AND DESIGN IN TELECOMMUNICATIONS

(M.Eng.Tele)

3 OCTOBER 2023
10:30 AM – 1:30 PM
(3 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 11 pages with 4 Questions only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please print all your answers in the Answer Booklet provided.

Question 1

(a) A system is described given by $y[n]=x^3[n]+3$. Determine if this system is

(i) Causal. [2 marks]

(ii) Memoryless. [2 marks]

(iii) Linear. [3 marks]

(b) A linear time-invariant (LTI) system is characterized by the following system function:

$$H(z)=\frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$$
$$=\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{2}{1-3z^{-1}}$$

(i) Obtain the linear constant coefficient difference equation of the LTI system. [2 marks]

(ii) Plot the region of convergence (ROC) if the LTI system is causal. [3 marks]

(iii) Plot the ROC if the LTI system is anti-causal. [3 marks]

(iv) Obtain the impulse response, $h[n]$, if the LTI system is stable. [2 marks]

(c) (i) Summarize the differences between the discrete-time Fourier transform (DTFT) and the discrete Fourier transform (DFT). [5 marks]

(ii) Figure Q1 shows a practical approach to performing spectral analysis on a continuous-time signal $g(t)$ by computing the DTFT, $G(e^{j\omega})$ of its discrete-time equivalent $g[n]$. The DTFT is usually approximated by the DFT operation. In this context, explain the difference between *high-density spectrum* and *high-resolution spectrum*. [3 marks]

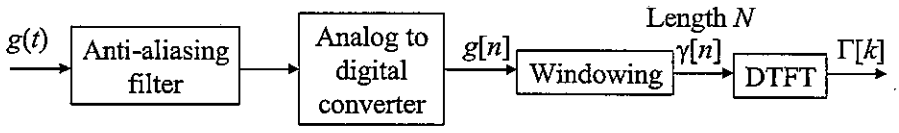

Figure Q1

Continued...

Question 2

(a) In the context of designing digital finite impulse response (FIR) filters, answer the following questions.

(i) Explain the rationale of truncating the impulse response of an ideal FIR digital filter. [2 marks]

(ii) With the aid of a diagram, explain the effects caused by this truncation. Suggest a method to minimize them. [3 marks]

(iii) When a smaller truncation window is used for truncating the impulse response of the ideal filter, explain the effect on the transition width and side-lope amplitude of the frequency response of the truncated filter. [3 marks]

(b) A signal is sampled at a frequency of 2 kHz, has a useful content from 0 to 200 Hz and is corrupted with noise from 400 to 500 Hz. Design a digital FIR low-pass filter based on a fixed window to attenuate the noise by at least 51 dB without affecting the useful content by more than 3 dB.

[Note: the impulse response of the ideal low pass filter is $h_D[n] = \frac{\sin(\omega_c n)}{n\pi}$] [7 marks]

(c) Briefly explain the concept of the bilinear transformation in designing digital IIR filters. Once you have obtained the digital transfer function $H(z)$, explain the rationale of the pre-warping technique in the bilinear transformation [6 marks]

(d) Perform the transpose operation to the digital filter structure shown in Figure Q2. [4 marks]

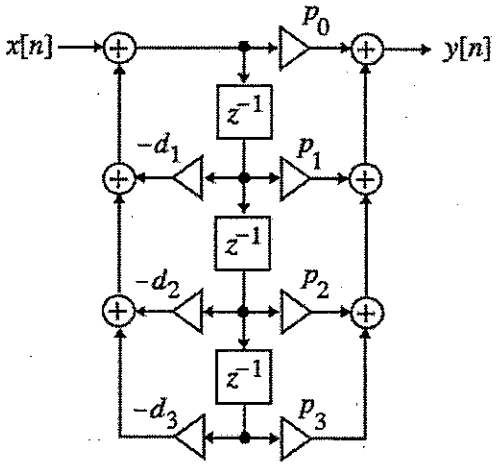

Figure Q2

Continued...

Question 3

(a) Consider a digital audio signal with a sampling rate of 44.1 kHz.

(i) Explain the concept of downsampling and how it is accomplished to reduce the data rate of the digital audio signal. [3 marks]

(ii) If the original audio signal contains frequencies up to 10 kHz, design a downsampling factor and calculate the new sampling rate to achieve a 50% reduction in data rate while still preserving all frequencies up to 20 kHz. [3 marks]

(b) (i) Briefly discuss the rationale for using multistage design and polyphase decomposition in multirate digital signal processing systems. [6 marks]

(ii) Consider the interpolator structure in Figure Q3. The filter $H(z)$ has a transfer function $H(z) = 3z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 3z^{-5} + z^{-6} + 2z^{-7} + 4z^{-8}$.

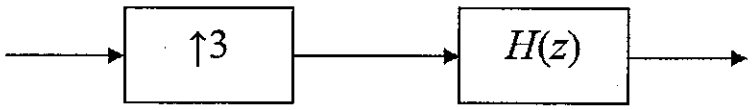

Figure Q3

The M-branch polyphase decomposition of a filter $H(z)$ can be expressed as: $H(z) = \sum_{k=0}^{M-1} z^{-k} E_k(z^M)$, where $E_k(z)$ are the subfilters. Derive the 3-branch polyphase decomposition of $H(z)$ for the above system. [7 marks]

(c) Subband coding of speech and audio signals can be accomplished using digital filter banks.

(i) Briefly describe this technique and explain how does exploit the frequency characteristics of speech and audio signals to achieve efficient compression. [3 marks]

(ii) Briefly describe an example of a subband coding system using a digital filter bank and describe the process of encoding and decoding. [3 marks]

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Question 4

(a) (i) Compare and contrast adaptive filters and static filters in terms of their applications and weight coefficients. [3 marks]

(ii) Explain the principle and key steps of the Least Mean Squares (LMS) adaptive algorithm used in adaptive filtering. [6 marks]

(iii) How does the step size parameter impact the convergence speed and tracking capabilities of the LMS algorithm? Discuss the trade-offs in selecting an appropriate step size. [4 marks]

(b) In the context of applications of digital signal processing, briefly describe identification and inverse modeling. Explain one example related to telecommunications for each. [6 marks]

(c) Figure Q4 depicts the transmission of data $u[n]$ through a communication channel. However, the transmitted data is subject to distortion due to inter-symbol interference, arising from the time-varying nature of the channel. To mitigate the effects of inter-symbol interference, an adaptive channel equalizer is necessary.

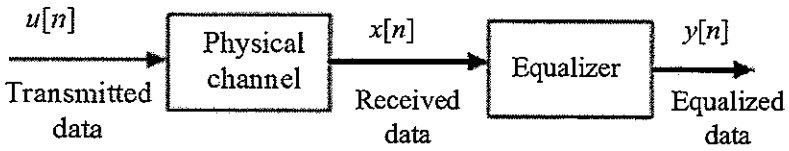

Figure Q4

(i) Modify the block diagram shown in Figure Q4 to illustrate the implementation details of an adaptive channel equalizer. Clearly label the equalizer input, output, error signal and training sequence. [3 marks]

(ii) Describe the working principle of the adaptive channel equalizer. [3 marks]

End of Questions

Appendix: Formula Sheet

The z-transform

Properties of the z-transform

Property	$x[n]$	$X(z)$	$\mathcal{R}_x$
Linearity	$ax[n]+by[n]$	$aX(z)+bY(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$
Time shifting	$x[n-m]$	$z^{-m}X(z)$	$\mathcal{R}_x$
Time reversal	$x[-n]$	$X(z^{-1})$	$1/\mathcal{R}_x$
Convolution	$x[n]*y[n]$	$X(z)Y(z)$	$\mathcal{R}_x \cap \mathcal{R}_y$

Common z-transform pairs

$x[n]$	$X(z)$	$\mathcal{R}_x$
$\delta[n]$	1	$\forall z$
$\delta[n-n_0]$	z^{-n_0}	Possibly $\forall z$
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

Closed-form Expression for Some useful Series

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a|<1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \qquad |a|<1$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

Continued...

FIR Filter Design

Ideal bandpass

$$h[n] = \frac{w_2}{\pi}\,\mathrm{sinc}\left(\frac{w_2(n-M/2)}{\pi}\right) - \frac{w_1}{\pi}\,\mathrm{sinc}\left(\frac{w_1(n-M/2)}{\pi}\right),$$
$$n = 0, 1, \ldots, M$$

Fixed windows

Window	Window function
Rectangular	$w[n] = 1$
Hann	$w[n] = 0.5 - 0.5\cos(2\pi n/M)$
Hamming	$w[n] = 0.54 - 0.46\cos(2\pi n/M)$
Blackman	$w[n] = 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M)$

Window	Passband ripple $20\log_{10}\delta_p$	Stopband attenuation $20\log_{10}\delta_s$	Transition width $\lvert w_p - w_s\rvert$
Rectangular	-13	-21	$1.8\pi/M$
Hann	-31	-44	$6.2\pi/M$
Hamming	-41	-53	$6.6\pi/M$
Blackman	-57	-74	$11\pi/M$

Kaiser window

$$w[n] = \frac{I_0\left(\beta(1-(n/\alpha-1)^2)^{0.5}\right)}{I_0(\beta)}$$

$$\beta = \begin{cases} 0.1102(A-8.7), & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0, & A < 21 \end{cases}$$

$$M = \begin{cases} (A-7.95)/(2.285\Delta w), & A \ge 21 \\ 5.655/\Delta w, & A < 21 \end{cases}$$

Optimal Filter Design (Parks-McClellan Algorithm) Estimated Filter Order

$$N = \frac{-20\log\sqrt{\delta_p\delta_s} - 13}{14.6\Delta f}$$

Continued...

CLASSIFICATION OF LINEAR-PHASE FIR SYSTEMS

	$h[n]$ symmetric: $h[n] = h[N-n]$	$h[n]$ antisymmetric: $h[n] = -h[N-n]$
N even	**Type I Linear Phase Filter** $H(e^{j\omega}) = e^{-jN\omega/2}\sum_{k=0}^{N/2} a[k]\cos(k\omega)$ $a[0] = h[N/2]$ $a[k] = 2h[(N/2)-k]$	**Type III Linear Phase Filter** $H(e^{j\omega}) = je^{-jN\omega/2}\sum_{k=1}^{N/2} c[k]\sin(k\omega)$ $c[k] = 2h[(N/2)-k]$
N odd	**Type II Linear Phase Filter** $H(e^{j\omega}) = e^{-jN\omega/2}\sum_{k=1}^{(N+1)/2} b[k]\cos((k-1/2)\omega)$ $b[k] = 2h[\frac{(N+1)}{2}-k]$	**Type IV Linear Phase Filter** $H(e^{j\omega}) = je^{-jN\omega/2}\sum_{k=1}^{(N+1)/2} d[k]\sin((k-1/2)\omega)$ $d[k] = 2h[\frac{(N+1)}{2}-k]$

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IIR Filter Design

Normalized Butterworth lowpass

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.000							
2	1.414	1.000						
3	2.000	2.000	1.000					
4	2.613	3.414	2.613	1.000				
5	3.236	5.236	5.236	3.236	1.000			
6	3.864	7.464	9.142	7.464	3.864	1.000		
7	4.494	10.10	14.59	14.59	10.10	4.494	1.000	
8	5.126	13.14	21.85	25.69	21.85	13.14	5.126	1.000

Filter order

$$d = \left(\frac{(1-\delta_p)^{-2} - 1}{\delta_s^{-2} - 1} \right)^{0.5}$$

$$k = \frac{\Omega_p}{\Omega_s}$$

Design	Filter order
Butterworth	$N \geq \dfrac{\log d}{\log k}$
Chebyshev I, II	$N \geq \dfrac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)}$
Elliptic	$N \geq \dfrac{\log(16/d^2)}{\log(2/u)}$ $u = \dfrac{1-(1-k^2)^{0.25}}{1+(1-k^2)^{0.25}}$

$$\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$$

Continued...

Frequency transformations

Target class	Transformation	Edge frequencies of target class
Highpass	$s \rightarrow \frac{\Omega_p \Omega'_p}{s}$	Ω'_p
Bandpass	$s \rightarrow \Omega_p \frac{s^2 + \Omega_u \Omega_l}{s(\Omega_u - \Omega_l)}$	Ω_l, Ω_u
Bandstop	$s \rightarrow \Omega_p \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$	Ω_l, Ω_u

Impulse invariance transformation

$$H_a(s) = \sum_{k=0}^{p-1} \frac{A_k}{s - s_k} \longrightarrow H(z) = \sum_{k=0}^{p-1} \frac{T_s A_k}{1 - e^{s_k T_s} z^{-1}}$$

Bilinear transformation

$$H(z) = H_a(s)\Big|_{s = \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}}}$$

$$\Omega = 2 \tan(w/2)/T_s$$

Continued...

Discrete-time Fourier Analysis

The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1 x_1[n] + A_2 x_2[n]$	$A_1 X_1[k] + A_2 X_2[k]$
Time shifting	$x[\langle n - n_0 \rangle_N]$	$X[k] W_N^{kn_0}$
Frequency shifting	$x[n] W_N^{-k_0 n}$	$X[\langle k - k_0 \rangle_N]$
Time reversal	$x[\langle -n \rangle_N]$	$X[\langle -k \rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k \rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k] Y[k]$
Modulation	$N x[n] y[n]$	$X[k] \circledast Y[k]$

End of Paper